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$$\mu \quad P(x) = 2x^2 + 3x + 1 \quad Q(x) = x^2 - 3.$$

$$\mu \quad P(Q(x)).$$

$$\mu \quad \alpha, \beta, \gamma, \delta, \varepsilon \in \mathbb{R} \quad : \quad \alpha x^8 + \beta x^6 + \gamma x^4 + \delta x^2 + \varepsilon = P(Q(x))$$

$$x \in \mathbb{R}.$$

$$\mu \quad A(x) = P(P(P(x))) + 6x.$$

$$\mu \quad (x) = x(2x +) + , \quad 1.$$

$$\text{i.} \quad | \geq 2 - | | .$$

$$\text{ii.} \quad , \in \mathbb{R} \quad (x) = 2x^2 + x \quad x \in \mathbb{R} .$$

$$\mu \quad \frac{Q(x) - x^2 + x}{P(x)} \quad \mu \quad \mu \quad \mu$$

$$\mu \quad \mu \quad .$$

$$\begin{aligned}) P(Q(x)) &= 2(x^2 - 3)^2 + 3(x^2 - 3) + 1 = 2(x^4 - 6x^2 + 9) + 3x^2 - 9 + 1 = \\ &= 2x^4 - 12x^2 + 18 + 3x^2 - 8 = 2x^4 - 9x^2 + 10 \end{aligned}$$

$$\begin{aligned}) \quad x \in \mathbb{R} \quad \alpha x^8 + \beta x^6 + \gamma x^4 + \delta x^2 + \varepsilon &= 2x^4 - 9x^2 + 10 \Leftrightarrow \\ = 0 \quad = 0 \quad = 2 \quad = -9 \quad = 10 \end{aligned}$$

$$) \quad P(-1) = 2(-1)^2 + 3(-1) + 1 = 2 - 3 + 1 = 0$$

$$A(-1) = P(P(P(-1))) + 6(-1) = P(P(0)) - 6 = P(1) - 6 = 6 - 6 = 0, \quad -1 \quad (x).$$

$$\begin{aligned}) \text{ i.} \quad 1 \quad (x) \quad (1) = 0 &\Leftrightarrow 2 + \quad + = 0 \Leftrightarrow \quad + = -2 \quad (1) \\ | + | \leq | + | &\Leftrightarrow 2 \leq | + | \Leftrightarrow | \geq 2 - | \quad | . \end{aligned}$$

$$\begin{aligned} \text{ii.} \quad (x) = x(2x + \quad) + \quad = 2x^2 + x + \quad . \quad (x) = 2x^2 + x \quad = 0, \\ (2) \quad = -2 \end{aligned}$$

$$\begin{aligned}) \quad \frac{Q(x) - x^2 + x}{P(x)} &= \frac{x^2 - 3 - x^2 + x}{2x^2 + 2x + x + 1} = \frac{x - 3}{2x(x+1) + (x+1)} = \frac{x-3}{(x+1)(2x+1)} \\ \frac{x-3}{(x+1)(2x+1)} &= \frac{\alpha}{x+1} + \frac{\beta}{2x+1}, \quad : \\ \frac{x-3}{(x+1)(2x+1)} &= \frac{\alpha(2x+1) + \beta(x+1)}{(x+1)(2x+1)} = \frac{2\alpha x + \alpha + \beta x + \beta}{(x+1)(2x+1)} = \frac{(2\alpha + \beta)x + \alpha + \beta}{(x+1)(2x+1)}. \\ x+1 \neq 0 &\Leftrightarrow x \neq -1 \quad 2x+1 \neq 0 \Leftrightarrow x \neq -\frac{1}{2}, \end{aligned}$$

$$x-3 = (2 + \quad)x + \quad \Leftrightarrow \begin{cases} 2 + \quad = 1 \\ + \quad = -3 \end{cases} \Leftrightarrow \begin{cases} 2 + \quad = 1 \\ = -3 - \end{cases} \Leftrightarrow \begin{cases} 2 - 3 - \quad = 1 \\ = -3 - \end{cases} \Leftrightarrow \begin{cases} \quad = 4 \\ \quad = -7 \end{cases},$$

$$\frac{x-3}{(x+1)(2x+1)} = \frac{4}{x+1} - \frac{7}{2x+1}.$$